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THE PERMEABILITY COEFFICIENT OF A POROUS MEDIUM SATURATED WITH A GAS OR LIQUID AFTER A CONFINED EXPLOSION

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UDC 543.222

Considerable scientific and practical interest attaches to the changes in infiltration properties of a medium produced by a confined explosion. One of the first attempts to determine the permeability coefficient in the region of such an explosion is to be found in [1]. Here we give results from processing experimental data obtained on a porous saturated medium after a confined explosion. The methods of examining the infiltration properties have been described in detail [2, 3]. The pressure difference and the fliud flow rate between different points in the medium were determined in the stationary state. These data are used as initial ones in solving the two-dimensional inverse problem for the permeability coefficient. The method of solving the problem is applied in processing experimental results obtained on a porous saturated medium after a confined explosion.

Experimental Data. A method analogous to that of [2, 3] was used in examining the infiltration properties of the medium after a confined explosion. The experimental explosions were performed in an artificially cemented medium having properties similar to those of real collectors and constituting a mixture of dressed sand, lime flour, and waterglass. The medium was placed in a cylindrical metal vessel of diameter 300 mm and height 350 mm. We used TEN charges of mass 0.4, 0.76, and 1.34 g. Each charge was placed at the middle of the model and was detonated from the center.

A comprehensive study was made of the mechanical effects in the high-porosity medium (m = 25%). The results provided an answer on whether there is any difference in infiltration parameters for monolithic and porous media when acted on by the explosion energy, and what is the difference in these properties if the explosion is performed in a medium in which the pores are filled with air at atmospheric pressure or with a liquid.

Tubes of diameter 3 mm were placed at various distances from the charge between the cavity and the periphery in the models to examine the changes produced by the explosion. The ends of the tubes were perforated and the opposite ends were connected to a measurement system. The tubes were placed in the horizontal or vertical plane of the charge. The model enclosed in the metal cylinder was hermetically sealed by flanges at the ends. Figure 1 shows the disposition of the tubes.

We determine the steady-state flow rate Q_i of air or kerosene and the corresponding pressure difference between a pair of tubes before and after explosion. The infiltration characteristic for the liquid-saturated medium was the ratio $\Gamma = Q_i/\Delta p_i$, where Q_i is the steady-state flow rate and $\Delta p_i = p_{i+1} - p_i$ is the pressure difference between a pair of tubes, while $i = 1, 2, \ldots, N$ represents the tube number. In a gas-saturated medium, Γ is defined by $\Gamma = Q/(p_{i+1}^2 - p_i^2)$.

The parameter change due to the explosion was evaluated from Γ/Γ_0, where Γ_0 is the characteristic before the explosion. Figures 2 and 3 give the results from the experiments.

Inverse Permeability Determination from Experimental Data. The pressure difference Δp_i between tubes is determined not only by the permeability of the medium between them but also by the properties of adjacent regions, as well as by the geometry of the model. It is therefore necessary to solve the inverse problem in order to determine the rational dependence of the permeability coefficient from the results.

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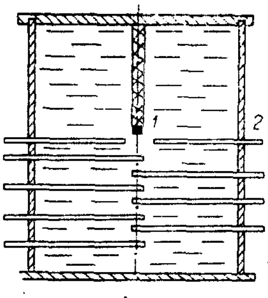

Fig. 1

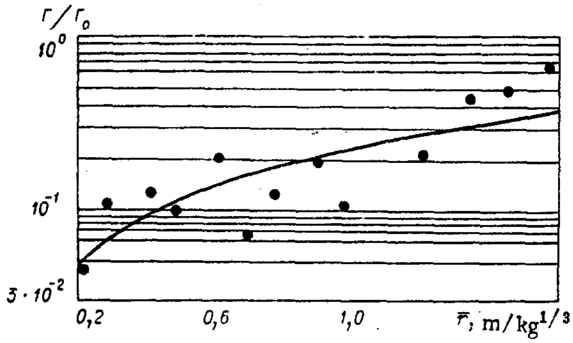

Fig. 2

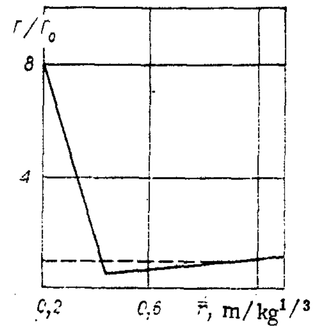

Fig. 3

Consider the fluid filtration in a medium bounded by a sphere of radius R. We assume that the distribution of the permeability coefficient within the sphere is spherically symmetrical, while the flow through the outer boundary is zero. We specify a source-antisource pair on some radius drawn from the center of the sphere and use the data characterizing Δp_i and Q_i for the flow between these points to determine the distribution k(r) of the permeability coefficient. The stationary process with this source-antisource system can be described in a spherical coordinate system with allowance for the axial symmetry as follows [4]:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 k(r)\frac{\partial u}{\partial r}\right)+\frac{k(r)}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial u}{\partial\theta}\right)=-A\mu Q\left[\delta(\mathbf{r}-\mathbf{r}_i)-\delta(\mathbf{r}-\mathbf{r}_{i+1})\right]; \qquad (1)$$

$$\partial u/\partial r|_{r=R}=0; \qquad (2)$$

$$u|_{r=0}<\infty, \qquad (3)$$

where r and θ are the radial and angular coordinates on the plane of the meridional section, R is the coordinate for the outer boundary of the spherical specimen, k(r) is the permeability, which is a function only of the radius, μ is the viscosity of the fluid, Q is the flow rate, the amount pumped through the system in unit time, and $\delta(\mathbf{r}-\mathbf{r}_i)$ is the source δ function.

Here one should bear in mind that a real source or sink has a finite characteristic size ρ, but to simplify the mathematical calculations they are usually replaced by point ones that provide the necessary flow Q and pressure p at the boundary.

System (1)-(3) describes the infiltration of a liquid or gas. In the case of a liquid, we have u = p and A = 1, while for a gas $u = p^2$ and A = 2, where p is the pressure of the fluid, p_i is the pressure at a point in the source, and p_{i+1} is the pressure at a point in the antisource. Condition (2) means that the outer boundary of the spherical specimen is impermeable, while condition (3) means that the pressure is bounded at the center of the sphere.

We assume that k(r) is unknown and attempt to recover if from information on Q_i and $\Delta p_{i,i+1}$; the following method is used: We rewrite (1) as

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial u}{\partial\theta}\right)=-\frac{1}{k(r)}A\mu Q\left[\delta(\mathbf{r}-\mathbf{r}_i)-\delta(\mathbf{r}-\mathbf{r}_{i+1})\right]-\frac{1}{k(r)}\frac{\partial k}{\partial r}\frac{\partial u}{\partial r}. \qquad (4)$$

In the steady state, $\partial u/\partial r$ is independent of time and is some function of the radius vector r, while on the left in (4) we have the Laplacian of the function u in the spherical coordinate system, where $\partial u/\partial\varphi=0$, so equation (4) with boundary condition (2) represents a second boundary-value problem for a sphere:

$$\Delta u = f(\mathbf{r}),\ \partial u/\partial r|_{r=R}=0, \qquad (5)$$

where

$$f(\mathbf{r})=\frac{1}{k(r)}A\mu Q\left[\delta(\mathbf{r}-\mathbf{r}_i)-\delta(\mathbf{r}-\mathbf{r}_{i+1})\right]-\frac{1}{k(r)}\frac{\partial k}{\partial r}\frac{\partial u}{\partial r}.$$

The solution to (5) can be obtained by the Green's function method [4]:

$$u(\mathbf{r})=-\iiint_{\Omega} f(\mathbf{r}_1)\,G(\mathbf{r}_1,\mathbf{r})\,d\mathbf{r}_1, \qquad (6)$$

where Ω is the region of integration $|r| \leqslant R$ and $G(r_i, r)$ is the Green's function.

The Green's function for the second boundary-value problem takes the form [5]

$$G(r_1, r) = \frac{1}{4\pi}\left\{-\frac{1}{|r_1 - r|} - \frac{R}{|r| r_0} - \frac{1}{R}\ln\frac{2R^2}{R^2 + |r| r_0 - |r||r_1|\cos\theta}\right\}, \tag{7}$$

where $r_0 = \sqrt{\left(\frac{R}{|r|}\right)^2 + |r_1|^2 - 2R\frac{|r_1|}{|r|}\cos\theta}$, $\cos\theta = \frac{(r_1, r)}{|r_1||r|}$.

We use (6) and (7) to get the final expression for u(r) at the arbitrary point r_j at which the pressure is measured:

$$u(r_j) = \frac{A\mu Q}{4\pi}\left[\frac{G(r_i, r_j)}{k(r_i)} - \frac{G(r_{i+1}, r_j)}{k(r_{i+1})} - I_0,\right. \tag{8}$$

where $I_0 = \frac{1}{4\pi}\iiint_\Omega \frac{\nabla k}{k(r_1)} \nabla u G(r_1, r_j)\, dr_1$.

The vectors r_i and r_{i+1} take the form

$$r_i = \{r_i,\ \theta = 0\};\ r_{i+1} = \{r_{i+1},\ \theta = 0\},$$

so subsequently the vectors oriented along the symmetry axis in the direction $\theta = 0$ would not be shown as vectors. The points of measurement for convenience also lie on the axis $\theta = 0$.

Then the pressure difference between points r_j and r_{j+1} can be put as

$$\Delta u_{j,j+1} = \frac{A\mu Q}{4\pi}\left\{[G(r_i, r_j) - G(r_i, r_{j+1})]\frac{1}{k(r_i)} + [G(r_{i+1}, r_{j+1}) - G(r_{i+1}, r_j)]\frac{1}{k(r_{i+1})}\right\} + I_j, \tag{9}$$

where $I_j = \frac{1}{4\pi}\iiint_\Omega \frac{\nabla k}{k(r_1)}\frac{\partial u}{\partial r}[G(r_1, r_j) - G(r_1, r_{j+1})]\, dr_1$.

As $G(r_i, r_j) \approx G(r_{i+1}, r_{j+1}) \approx 1/(4\pi\rho)$, where ρ is the characteristic size of the source, and $G(r_i, r_{j+1}) \approx G(r_{i+1}, r_j) \approx 1/(4\pi\Delta)$, where Δ is the distance between the source and antisource, and $\Delta >> \rho$, we have from (9) the recurrence formula

$$\frac{1}{k(r_{i+1})} = -\frac{1}{k(r_i)} + \frac{\rho}{A\mu Q}[4\pi A\Delta u_{j,j+1} + I_j]. \tag{10}$$

Then if the permeability is known before the explosion and if we assume that the characteristics are only slightly altered after the explosion in the peripheral region, we can begin with the most remote point and calculate $k(r_i)$ for all the points at which the sources lie. Clearly, one can calculate the permeability from (10) only by successive approximation, since to calculate the integral I one needs to know the distribution of k(r) and the value of the derivative $\partial u(r)/\partial r$ defined by it.

We used the following calculation algorithm. In the zeroth approximation, we put $I_j^{(0)} = 0$, and from (10) we calculated k_i^0.

The values for k_i^0 at the discrete points were used to $k^{(0)}(r)$ as polynomials by least squares. Then the $k^{(0)}(r)$ relationship was substituted into (1) and the series of direct two-dimensional problems of (1)-(3) was solved for the various source-antisource pairs. The problem of (1)-(3) was handled numerically by the method of [6] using longitudinal-transverse fitting in the r and θ coordinates. The numerical method of solving the problems has been described in [7].

The distributions u(r) obtained in this way were used to calculate $I_j^{(1)}$: The $I_j^{(1)}$ found in the first approximation enabled one to calculate the k_i in the next one. The successive approximations were performed in a similar fashion, the convergence criterion being

$$\max|I_j^{(l+1)} - I_j^{(l)}| < \varepsilon,$$

where ε is a given small quantity and l is the number of the approximation.

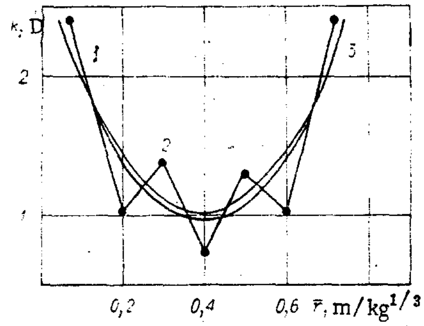

Fig. 4

Inverse-Problem Solution. We consider the method of solving the inverse problem with a known dimensionless model distribution for the permeability $k_0(r) = 1 + 12.5(0.5 - r/R)^2$; for this distribution, we solve the series of two-dimensional problems of (1)-(3) with various source-antisource pairs, which enable us to determine the corresponding pressure difference for a given value of Q. These differences were used as input data for recovering the k(r) profile. It was assumed that $A\mu Q = 1$, $R = 1$, and the sources were arranged with a step of $\Delta = 0{,}1R$, with the characteristic size ρ of a source determined by the dimensions of the computing cell. In the calculations we used a net with uniform steps in r and θ. The number of nominal points in the r and θ coordinates were equal and were 21 and 41 in different treatments.

The characteristic source size decreases towards the center of the sphere, so the dimensions of a computing cell are dependent on the distance to the center:

$$\rho_i \sim r_i^{2/3}. \tag{11}$$

The calibration was performed by calculating $\Delta u_{i,i+1}$ for k = constant followed by calculation of ρ_i from (10), and this gave the dependence of ρ_i on radius coincident with (11). In using (10), we either specify the exact value of $k(r_N)$ in the peripheral zone or assume that $k(r_N) = k(r_{N-1})$. The error introduced by equating the values of k(r) at the two extreme points results in a certain increase in the error in determining k(r), but the calculations showed that this error did not exceed 5% in this model treatment. Figure 4 shows the results from solving the inverse problem (curve 1 is the true distribution of the permeability coefficient, 2 is the zeroth approximation $k^{(0)}(r_i)(I_j^{(0)} = 0)$ and 3 is the approximation to the relationship with polynomials provided by least squares). Even the zeroth approximation (curve 2) correctly reflects the qualitative behavior of $k_0(r)$, while quantitatively the difference from the true solution is not more than 7%. The next approximation gives essentially the exact solution. The convergence is rapid because the $I_j^{(l)}$ are small by comparison with $4\pi A\Delta u_{j,j+1}$, since the integral I_j contains the logarithmic derivative of Δk/k, and so the integral I_j may be comparable with $4\pi A\Delta u_{j,j+1}$ only when k(r) varies rapidly in the region of definition.

Curve 2 is of sawtooth character, which is due to the discrete character of the specification of $\Delta u_{j,j+1}$; the amplitude of the oscillations decreases as the number of points increases. When the $\Delta u_{j,j+1}$ are specified discretely, there is no information on the behavior of k(r) between the measurement points, and therefore physical meaning attaches only to the average value of the permeability between the measurement points. For this reason it is necessary to average $k^{(l)}(r_i)$. In this study, this operation was realized by approximating the $k^{(l)}(r)$ curves with the polynomials.

This method of determining the permeability was used to process the experimental data obtained with confined explosions in the porous medium. In the calculations it was assumed that $\rho = (4-5)\cdot 10^{-1}\,\mathrm{cm}$, $\Delta_{i,i+1} = 2\,\mathrm{cm}$.

In principle, it is necessary to process each experiment separately and then average the set of k(r) distributions. However, it was not possible to obtain a continuous sequence of measurements between adjacent tubes in all the experiments, although this is necessary in order to use (10). Therefore, it is desirable to start by performing a statistical processing of the measurements $Q_{i,i+1}$, $\Delta u_{i,i+1}$ and coordinates r_j, r_{j+1}, and then to use the average data to calculate k(r). Figure 5 shows the results from such processing for a water-saturated medium with background permeability k_b = 0.4 D, and Fig. 6 shows the same for a gas-

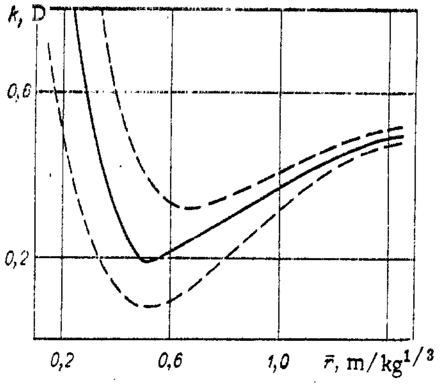

Fig. 5

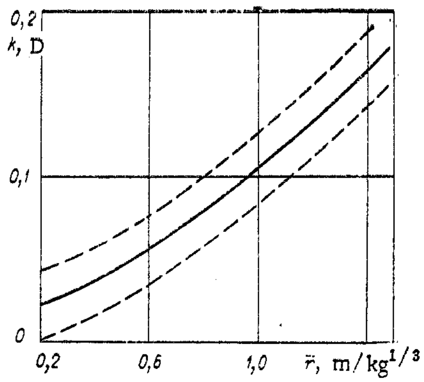

Fig. 6

saturated medium with $k_b = 0.32$ D. The initial porosities were the same in both cases (m = 25%). The broken lines in Figs. 5 and 6 show the confidence limits for the k(r) curves arising in the main from the statistical spread in Q_i and Δp_i.

These curves show that the k(r) for a water-saturated medium after explosion qualitatively coincides with the corresponding Γ/Γ_0 relationship (Fig. 3). As regards k(r) in a gas-saturated medium after explosion, it differs from the corresponding Γ/Γ_0 relation (Fig. 2) in being convex downwards. There is a tendency for the permeability in the region of the cavity to fall after the explosion in the saturated medium in the range $1.5a < r < 10a$, where a is the radius of the cavity. However, while the fall in the permeability is very substantial for the gas-saturated medium (at the minimum point, the permeability is reduced by an order of magnitude by comparison with k_b before the explosion), $k(r)/k_b \geqslant 1.2$ for the water-saturated case.

The main error in recovering the k(r) curve is determined by the spread in Γ. A second major factor that introduces error into determining k(r) is the inaccuracy in determining $\rho_i = |r_j - r_i|$, which in our experiments characterizes the effective source size. We see from (10) that if all the sources are of the same effective size, the error in determining ρ affects only the normalization of k(r), and has no influence on the r dependence. If the sources differ in effective size, error in determining the sizes may also affect the accuracy in determining the radial k(r) dependence. Therefore, it is desirable to measure the pressure difference not between the tubes through which the fluid is pumped but between tubes placed near them. In that case, $\rho_i = |r_j - r_i|$ will be larger and the relative error in determining it is reduced.

The geometry of the real model differs from the spherical one assumed in solving the inverse problem, but the height of the cylindrical model was approximately twice the radius of the base, while the flow speed near the corner points tends to zero, as calculations show. Therefore, the difference from spherical symmetry is slight, which enables one to handle two-dimensional direct problems instead of three-dimensional ones. The errors in determining the $\Delta p_{i,i+1}$ and Q_i were 3%. These data were used with the error associated with the spread in Γ/Γ_0 to calculate the confidence limits for k(r).

This study thus shows that a confined explosion in a porous (m = 25%) saturated medium reduces the permeability in the range $1.5a < r < 10a$; the most substantial reduction occurs after explosion in a gas-saturated medium, with k(r) at the minimum reduced by an order of magnitude.

The permeability reduction is not so substantial in a water-saturated medium: by less than a factor 2. Also, in that medium k(r) near the cavity exceeds k_b and the distribution is not monotonic.

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PROPAGATION OF NONLINEAR COMPRESSION PULSES IN GRANULAR MEDIA

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UDC 624.131 + 532.215 + 534.22

The study of mechanics of a granular medium is of substantial interest, both scientifically and for the solution of applied problems. Such materials are, for example, good buffers for shock loads. Their study is important for the development of processes of the pulse deformation of several porous materials. A review of studies of small deformations and elastic wave propagation in these media was carried out in [1] on the basis of discrete models. The structure of a stationary shock wave was analyzed in [2] as a function of its amplitude.

1. Statement of the Problem. The problem of nonstationary, nonlinear perturbations in one-dimensional granular media is stated in the present paper on the basis of the well-known interaction between neighboring granules.

As an interaction law we choose the Hertz law [3]

$$F=\frac{2E}{3(1-\nu^2)}\left(\frac{R_1R_2}{R_1+R_2}\right)^{1/2}\{(R_1+R_2)-(x_2-x_1)\}^{3/2}, \tag{1.1}$$

where F is the compression force of granules, E is the Young modulus of their material, R_1 and R_2 are radii, ν is the Poisson coefficient, and x_1 and x_2 are the coordinates of spherical granules ($x_2 > x_1$).

It is necessary to point out that a dependence of the form $\delta^{3/2}$, where δ is the closest approach of particle centers, is valid not only for spheres, but also for contacts of other finite bodies [3]. Interestingly, it is only due to the finite particle sizes of a linearly elastic material constituting the granular medium that its behavior has a nonlinearly elastic character.

The use of the static Hertz law in solving dynamic problems implies the following restrictions: 1) the maximum stress achieved at the center of the contact must be less than the elastic limit; 2) the sizes of the contact surface are much smaller than the radii of curvature of each particle; and 3) the characteristic times of the problem τ are much longer than the oscillation period of the basic shape for the elastic sphere T

$$\tau \gg T \approx 2.5R/c_1,$$

where c_1 is the velocity of sound in the sphere material.

Conditions 1-3 restrict the mass velocities of the medium to quantities of the order of several meters per second for metallic particles with radii in the interval 1-5 mm. Dissipation processes are not taken into account at the present stage of the study.

For numerical study of perturbation propagation processes in a one-dimensional chain of spherical particles with arbitrary radii R_i the second order equations of motion were reduced to a first order system of equations:

$$\dot{x}_i = F_i(\mathbf{x}),\ \mathbf{x}=(x_1, x_2, \ldots, x_{2N}),\ i=1, \ldots, 2N, \tag{1.2}$$
$$F_i(\mathbf{x}) = \varphi_i(\mathbf{x}) - \psi_i(\mathbf{x}),\ i=1, \ldots, N,$$

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